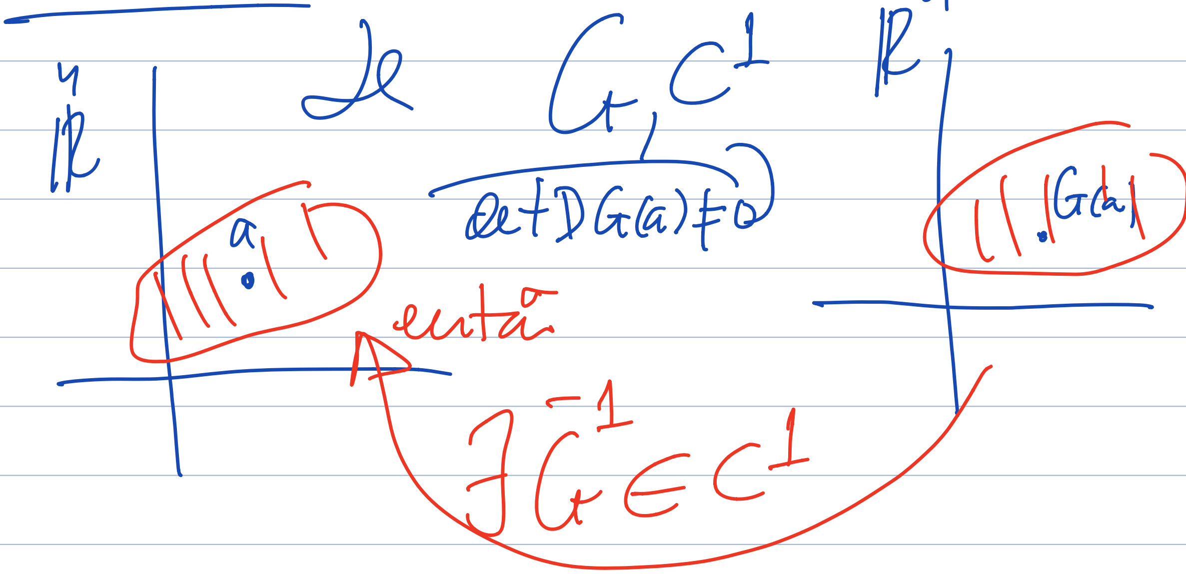
CDI-II — Prática 5/4/21

Ficha 8 – Inversa, Implícita

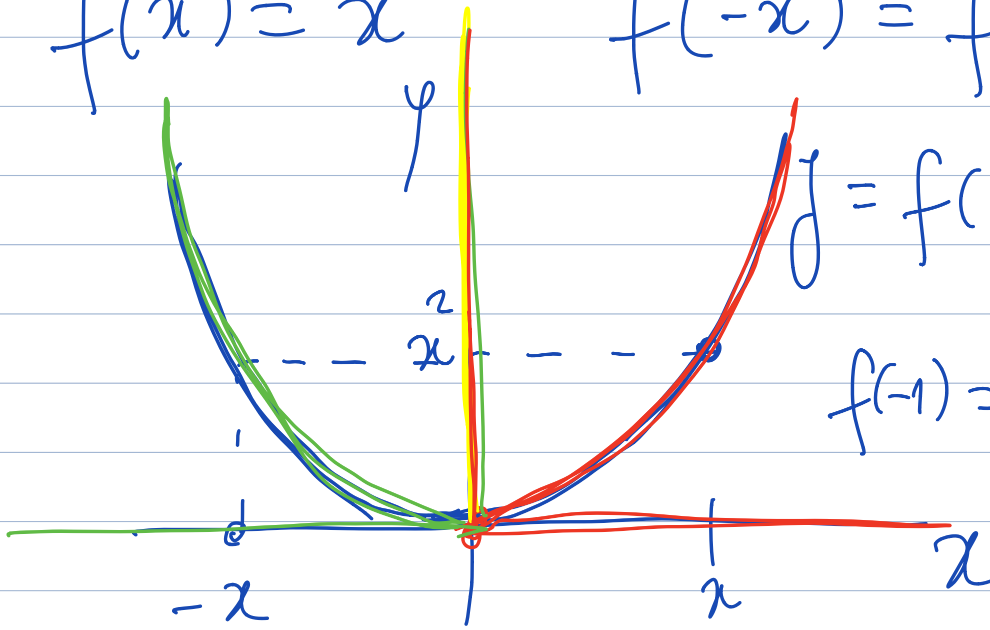
Inversa:



T. F. Composta

$$DG^{-1}(G(a)) = (DG(a))^{-1}$$

$f(x) = x^2$ $\quad$ $f(-x) = f(x)$



$y = f(x) = x^2$

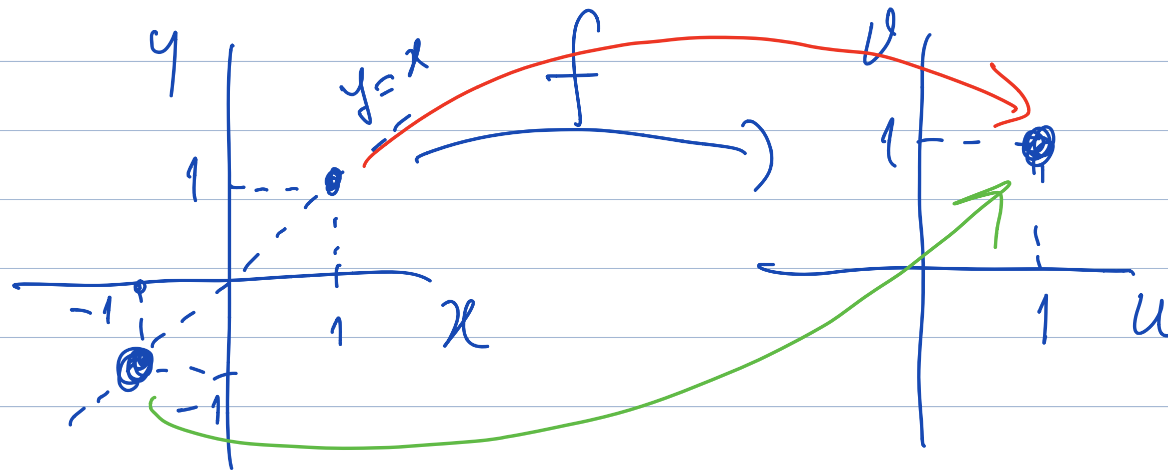
$f(-1) = f(1) = 1$

$f: \mathbb{R} \to \mathbb{R}$ não é injectiva (não tem inversa).

Se $\boxed{x > 0}$, $y = x^2 \Leftrightarrow x = +\sqrt{y}$

Se $x < 0$, $y = x^2 \Leftrightarrow x = -\sqrt{y}$

1-a) $f(x,y) = \left(xy, \frac{y}{x}\right) \quad x \neq 0$



$f(x,y) = (u,v) = \left(xy, \frac{y}{x}\right)$

$f(x,x) = (x^2, 1)$

$\underbrace{f(-1,-1) = f(1,1) = (1,1)}_{\text{?}}$

1-b) $f(x,y) = (u,v)$

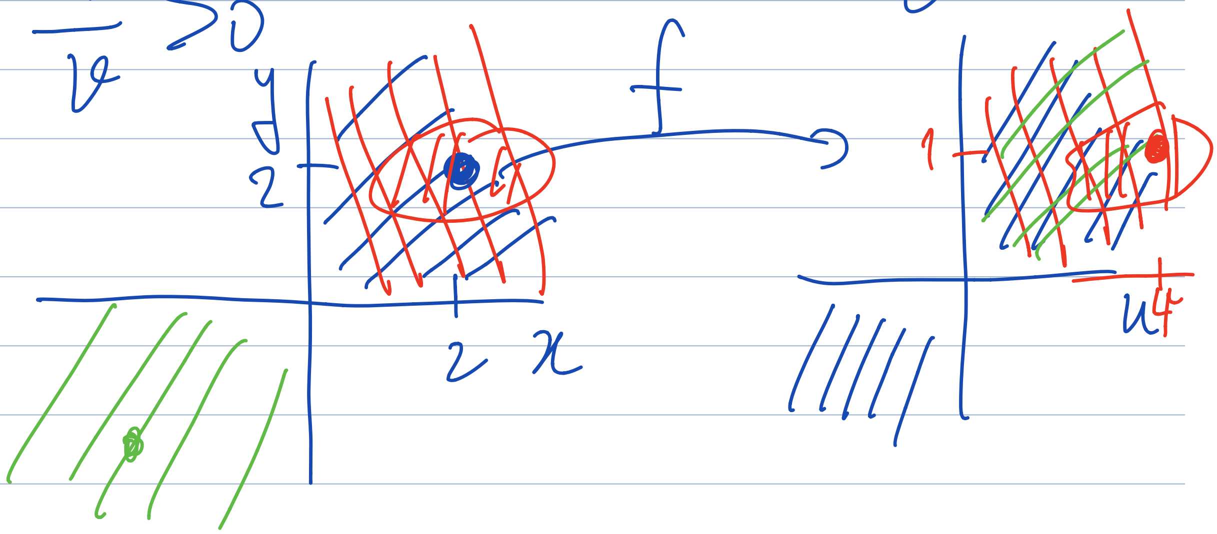
$$x \neq 0 \quad \begin{cases} u = xy \\ v = \dfrac{y}{x} \end{cases} \Leftrightarrow \begin{cases} u = x^2 v \\ xv = y \end{cases} \Leftrightarrow$$

$$\begin{cases} x = \sqrt{\dfrac{u}{v}} \\ y = \sqrt{\dfrac{u}{v}}\, v = \sqrt{uv} \end{cases} \vee \begin{cases} x = -\sqrt{\dfrac{u}{v}} \\ y = -\sqrt{uv} \end{cases}$$

$$\Downarrow$$

$$uv > 0$$

$$\frac{u}{v} > 0$$



1-c) $\det Df(2,2) \neq 0$ ✓ ?

$$Df(2,2) = \begin{bmatrix} y & x \\ -\dfrac{y}{x^2} & \dfrac{1}{x} \end{bmatrix}_{(2,2)}$$

$$= \begin{bmatrix} 2 & 2 \\ -\dfrac{1}{2} & \dfrac{1}{2} \end{bmatrix}$$

$\det Df(2,2) = 1+1 = 2 \neq 0$

$\Rightarrow$ f tem inversa local em $(2,2)$ e $(4,1) = f(2,2)$

1-d) $(4,1) = f(2,2)$

$$Df^{-1}(4,1) = \left(Df(2,2)\right)^{-1}$$

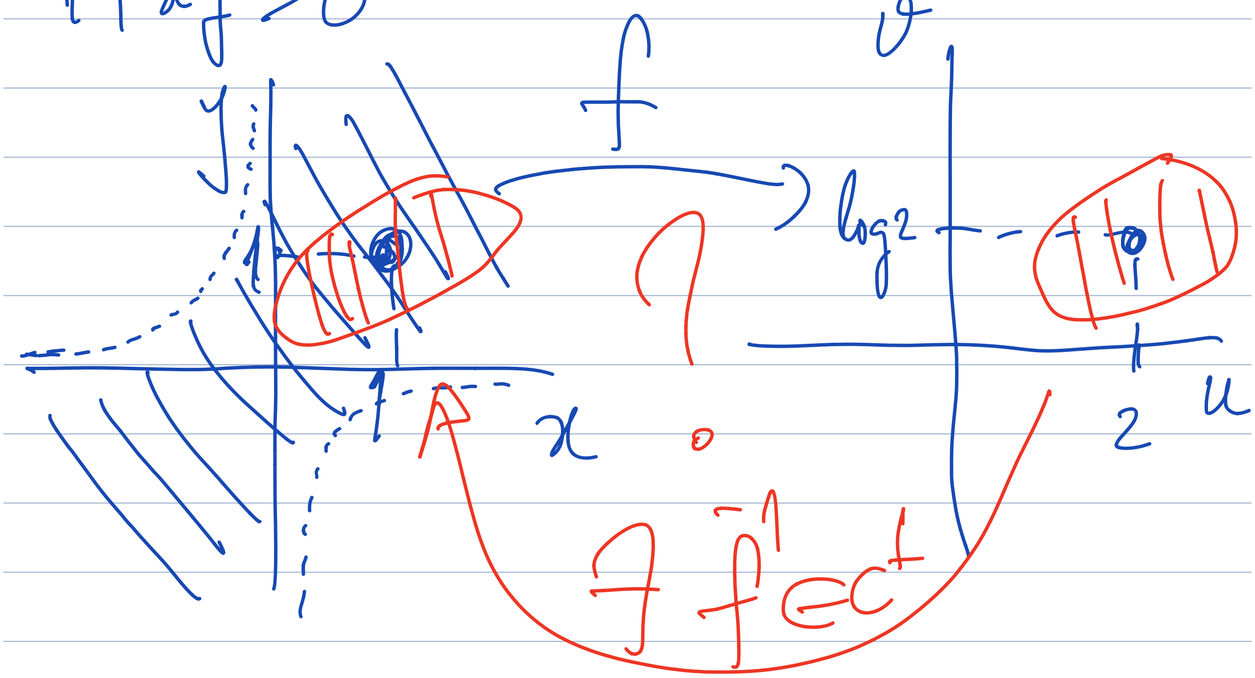
$$= \begin{bmatrix} 2 & 2 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}^{-1}$$

$$= \frac{1}{2} \begin{bmatrix} \frac{1}{2} & -2 \\ \frac{1}{2} & 2 \end{bmatrix} \quad \checkmark$$

2 — $(u,v) = f(x,y)$

$(x,y) = f^{-1}(u,v)$ $= \left(x+y+\operatorname{sen}(x-y),\ 1+\log(1+xy)-x\right)$

$1+xy>0$



$$Df(1,1) = \begin{bmatrix} 1+\cos(x-y) & 1-\cos(x-y) \\ -1+\dfrac{y}{1+xy} & \dfrac{x}{1+xy} \end{bmatrix}_{(1,1)} = \begin{bmatrix} 2 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$\det Df(1,1) = 1 \neq 0$

$$Df(x,y)_{(1,1)} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}_{\substack{(x,y)\\(1,1)}}$$

$$Df^{-1}(u,v)_{(2,\log 2)} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \boxed{\frac{\partial y}{\partial v}} \end{bmatrix}_{\substack{(u,v)\\(2,\log 2)}}$$

$$= \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}^{-1}_{\substack{(x,y)\\(1,1)}}$$

$$Df^{-1}(2,\log 2) = \begin{bmatrix} 2 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & \boxed{2} \end{bmatrix}$$

$$\frac{\partial y}{\partial v}(2,\log 2)$$

Implícita:

1) $F : \mathbb{R}^n \to \mathbb{R}^m$, $m < n$, C^1

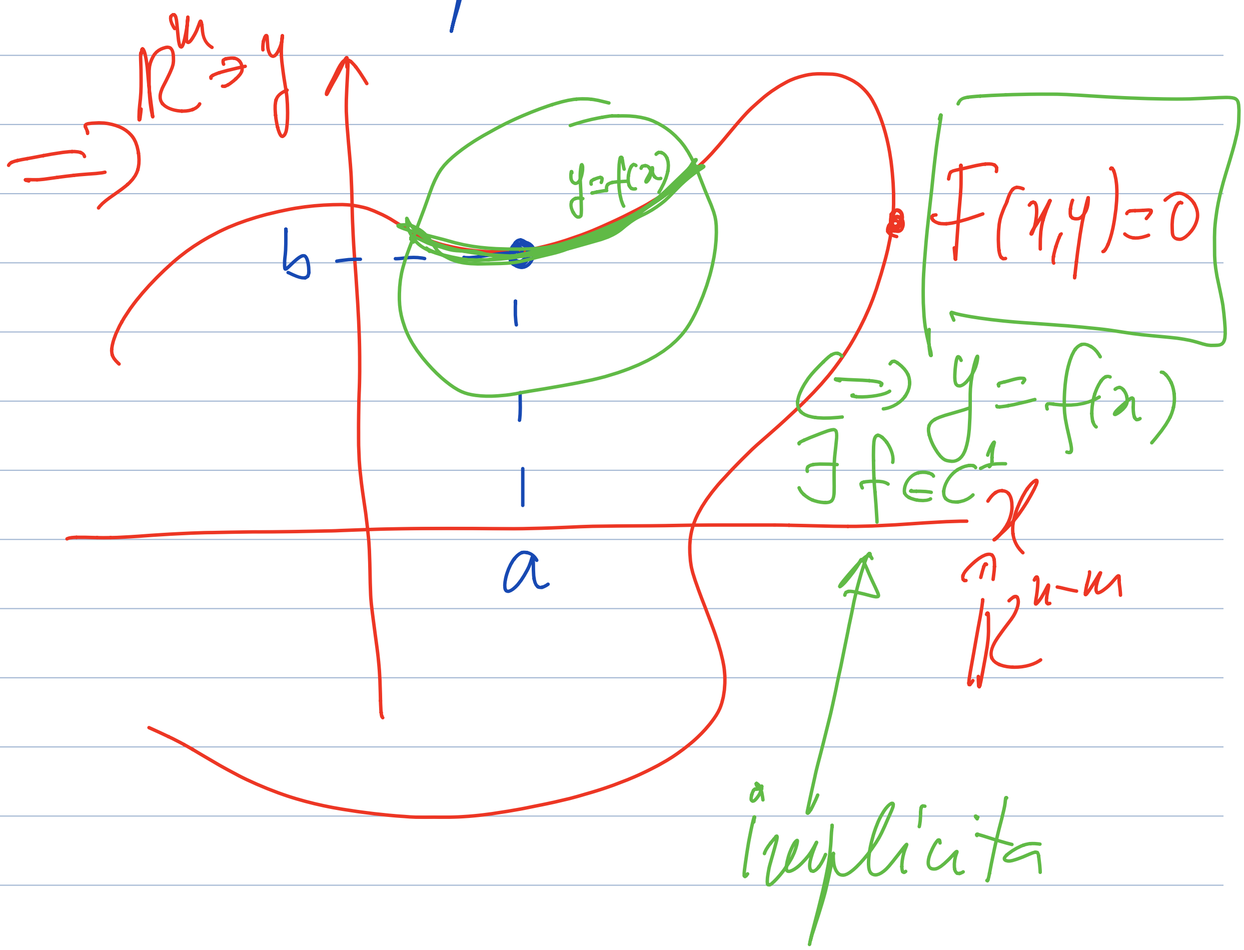
$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^m$

(x, y)

livres dependentes

2) $\exists (a,b)$ $F(a,b) = 0$

3) $\det D_y F(a,b) \neq 0$



R. Cadeic : $\quad y = f(x)$

$$F(x, f(x)) = 0$$

$$D_x F(a,b) + \underbrace{D_y F(a,b)}_{\det \neq 0} \boxed{Df(a)} = 0$$

$$Df(a) = -\left(D_y F(a,b)\right)^{-1} D_x F(a,b)$$

4— $F(x,y,z)=0$ $\quad F(0,0,1)=0$

$F:\mathbb{R}^3\to\mathbb{R},\ C^1$

$$DF(x,y,z)=\begin{bmatrix}\frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \boxed{\frac{\partial F}{\partial z}}\end{bmatrix}_{(x,y,z)}$$

$\neq 0 \quad (0,0,1)$

$$DF(0,0,1)=\begin{bmatrix}0 & 0 & \boxed{2}\end{bmatrix}$$

$\neq 0$

$$\boxed{z=z(x,y),\ C^1}$$

$\frac{\partial z}{\partial x}$?

$$F(x,y,z(x,y)) = 0$$

$$\underbrace{\frac{\partial F}{\partial x}(\overset{0}{x},\overset{0}{y},\overset{1}{z(x,y)})}_{0} + \underbrace{\frac{\partial F}{\partial z}(\overset{0}{x},\overset{0}{y},\overset{1}{z(x,y)})}_{2 \neq 0} \boxed{\frac{\partial z}{\partial x}(\overset{0}{x},\overset{0}{y})} = 0$$

$$\frac{\partial z}{\partial x}(0,0) = -\frac{0}{2} = 0.$$

Derivar novamente, mas em y. (Regra de cadeia)

etc

$$5-\begin{cases} F_1(x,y,z)=0 \\ F_2(x,y,z)=0 \end{cases}$$

$F:\mathbb{R}^3 \longrightarrow \mathbb{R}^2$, C^1 ✓

$F(0,1,0) = (0,0)$ ✓

$f: I \longrightarrow \mathbb{R}^2$

$\boxed{x \longrightarrow (y(x), z(x)) \quad ?}$

$y \longrightarrow (x(y), z(y)) \quad ?$

$\boxed{z \longrightarrow (x(z), y(z)) \quad ?}$

$$DF(0,1,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$$

$\det \neq 0$
$x(y), z(y)$

$\det \neq 0$
$x(z), y(z)$

$\det \neq 0$
$y(x), z(x)$

$$DF(0,1,0) = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$\det = -2 \neq 0$ $\quad$ $\det = 2 \neq 0$

$x = x(z)$
$y = y(z)$ $\quad C^1.$

$$
\begin{cases}
F_1\big(x(z), y(z), z\big) = 0 \\
F_2\big(x(z), y(z), z\big) = 0
\end{cases}
$$

$$
\begin{cases}
\overset{0}{\dfrac{\partial F_1}{\partial x}} \dfrac{dx}{dz} + \overset{2}{\dfrac{\partial F_1}{\partial y}} \dfrac{dy}{dz} + \overset{0}{\dfrac{\partial F_1}{\partial z}} = 0 \\[2ex]
\underset{1}{\dfrac{\partial F_2}{\partial x}} \dfrac{dx}{dz} + \underset{2}{\dfrac{\partial F_2}{\partial y}} \dfrac{dy}{dz} + \underset{1}{\dfrac{\partial F_2}{\partial z}} = 0
\end{cases}
$$

etc ...